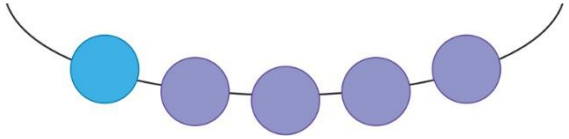
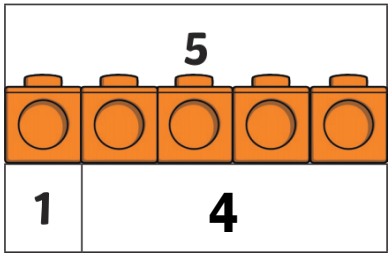
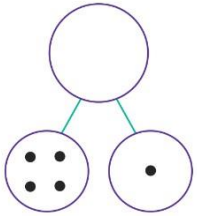
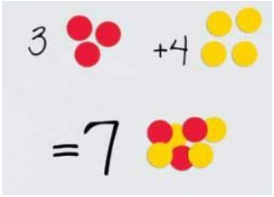
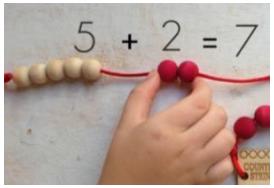
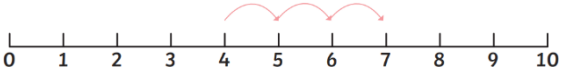
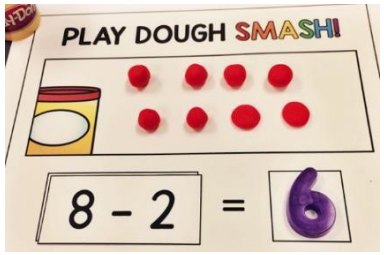
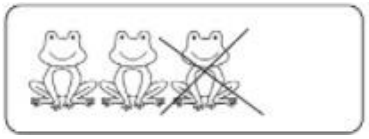
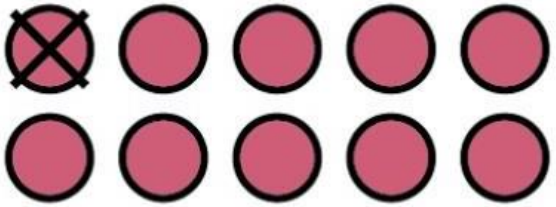
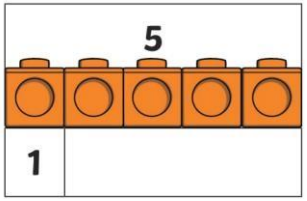
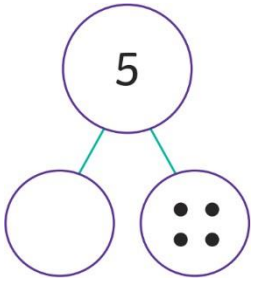
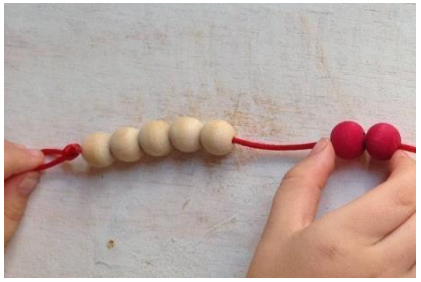
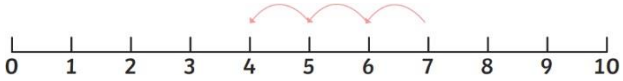
EYFS: Addition

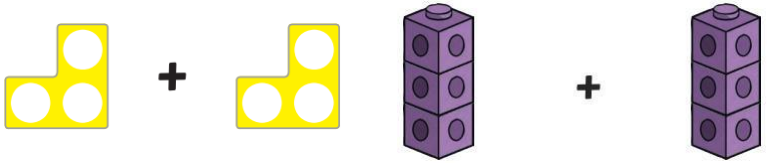
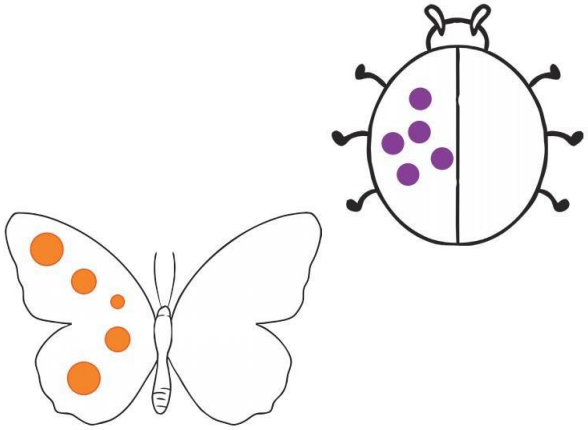
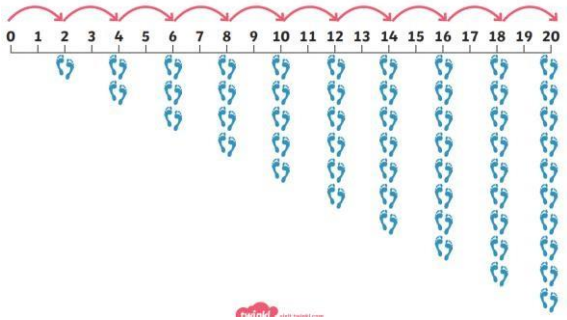
Strategy	Concrete	Pictorial	Abstract
Number bonds to 5.	Use physical apparatus/objects such as counters (five frame) or multi-link to make/ combine two parts together to make a whole. It is important to use this language. **2 + 3 = 5**  **1 + 4 = 5** 	Use pictures to add two numbers together as a group of.  **4 + 1 = 5** Use part, part whole models/bar model to show number bonds to 5.  **4 + 1 = 5**	Number sentence to 5: **3 + 2 = 5** **5 = 4 + 1** Equal symbol should be presented at the beginning and end of the number sentence to reinforce understanding of equals meaning same as/balance.
Counting on (starting with the largest number).	Use practical apparatus to make the largest number and then add on the remaining amount through counting on.   	Use a number line, starting with the largest number and counting on.  This can also be done using fingers/putting largest number in head and counting on. From using a part, whole model, demonstrate that numbers can be added in any order (commutative) **however** it is more efficient to begin with the largest number.	Number sentence: **7 + 4 = 11** Reorder the number sentence: **3 + 15 =** **15 + 3 = 18**

EYFS: Subtraction

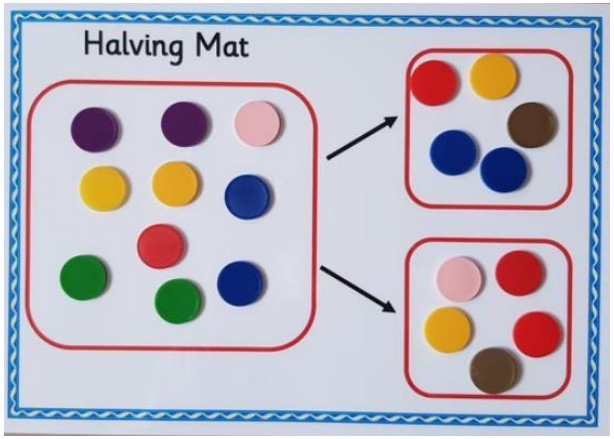
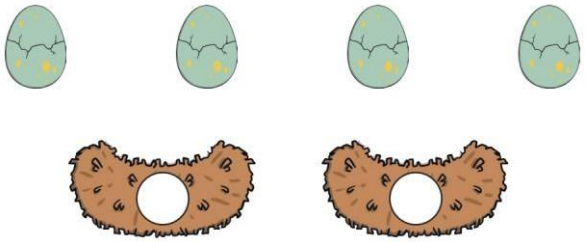
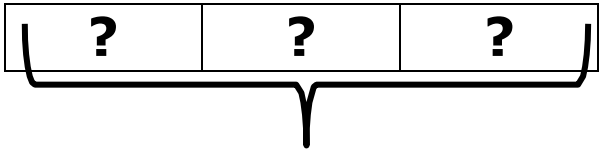
Strategy	Concrete	Pictorial	Abstract
Subtracting ones.	Use physical objects to show subtraction of ones.   3 – 1 =	Draw total amount of objects. Cross out number being subtracted  10 – 1 =	Number sentence: **13 – 1 = 12** **7 = 9 – 2** **Equal symbol should be presented at the beginning and end of the number sentence to reinforce understanding of equals meaning same as/balance.**
Number bonds to 5.	Use physical objects/apparatus such as counters (five frame) or multi-link to make a whole (5) and take away a part. It is important to use this language.  **5 – 3 = 2**	Use pictures, part, whole model and bar model to take away from a group of 5.  **5 – 1 = 4**  **5 – 4 = 1**	Number sentences: **5 – 4 = 1** **5 = 5 - 0**

Counting backwards.	Use practical apparatus to subtract by making the largest number in the number sentence and counting backwards. Bead String: Move the beads along the string, counting backwards in ones.  **7 – 2 = 5** Counters/Cubes/Objects: Move the objects away, counting backwards in ones.	Use a number line or number track to count backwards, starting with the largest number and counting backwards in jumps of ones. 7 - 3 = 4  This can also be done using fingers/putting largest number in head and counting backwards.	Number sentence: 7 - 3 = 4 Mental Calculation: **9 – 4 = 5** **Put 9 in your head and count back 4. What number have you landed on?**

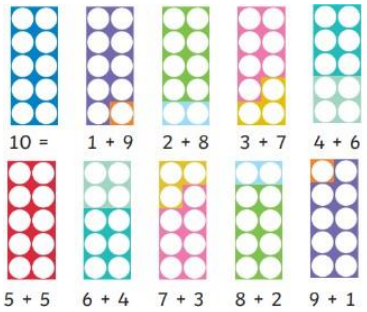
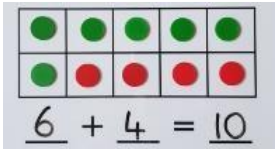
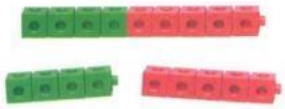
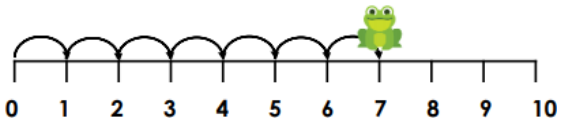
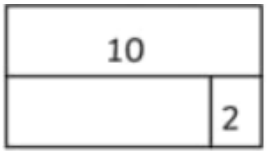
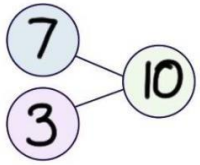
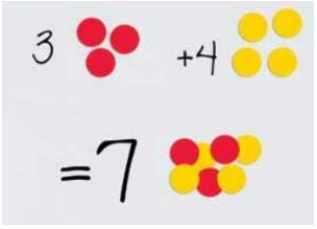
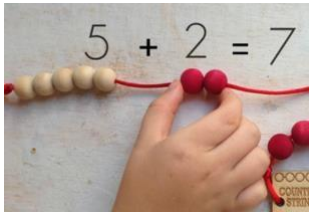
EYFS: Multiplication

Strategy	Concrete	Pictorial	Abstract
Doubling.	Use physical apparatus/objects such as counters or multi-link to make one group/lot.  Double the amount to make two groups/lots and count how many there are in total. 	Draw the amount (one group/lot).  Draw two lots of the amount and count the total.	**Number Sentence:** **4 + 4 = 8**
Counting in multiples.	Use practical apparatus/objects to count on in 2's. N.B. Counting backwards and forwards in 1's must be secure. 	Count on using a number line or number track. 	**Number Sequence:** **2, 4, 6, 8, 10**

EYFS: Division

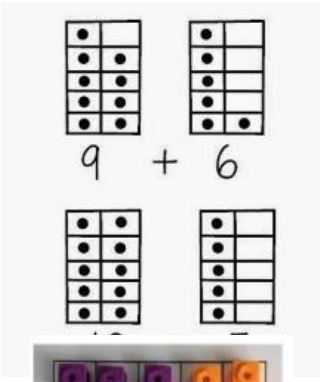
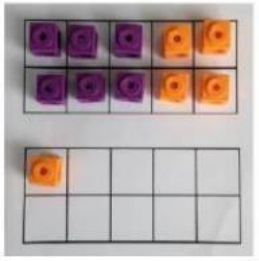
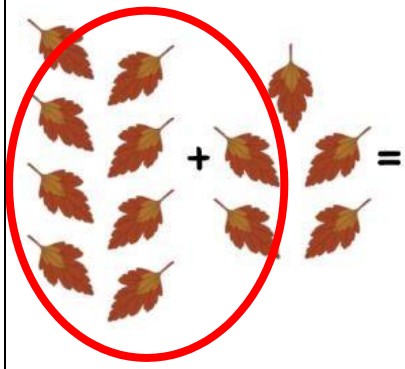
Strategy	Concrete	Pictorial	Abstract
Sharing into equal groups.	Use physical apparatus/objects such as counters or multi-link share an amount into equal groups.  Introduce halving as sharing into 2 equal groups. 	Represent sharing into equal groups pictorially through drawing sharing an amount into equal groups. **4 ÷ 2 = 2**  **6 ÷ 3 = 2**  **6**	**Number Sentence:** **6 ÷ 3 = 2** **Half of 10 = 5**

Year 1: Addition

Strategy	Concrete	Pictorial	Abstract
Number bonds to 5 and 10.	Use counters (ten frame), numicon or multi-link to make/ combine two parts together to make a whole. It is important to use this language.   	Use pictures to add two numbers together as a group of 5 or 10.   Use part, part whole models/bar model to show number bonds to 5/10.  	Number sentence to 5/10: **3 + 2 = 5** **5 = 4 + 1** **10 = 8 + 2** **7 + 3 = 10** Equal symbol should be presented at the beginning and end of the number sentence to reinforce understanding of equals meaning same as/balance.
Counting on (starting with the largest number).	Use practical apparatus to make the largest number and then add on the remaining amount through counting on.  	Use a number line, starting with the largest number and counting on.  This can also be done using fingers/putting largest number in head and counting on. From using a part, whole model, demonstrate that numbers can be added in any order (commutative) **<u>however</u>** it is more efficient to begin with the largest number.	Number sentence: **7 + 4 = 11** Reorder the number sentence: **3 + 15 =** **15 + 3 = 18**

<table>
<tr>
<td>Regrouping to make groups of 10.</td>
<td>Use counters and a ten frame to show number sentence starting with the largest number.

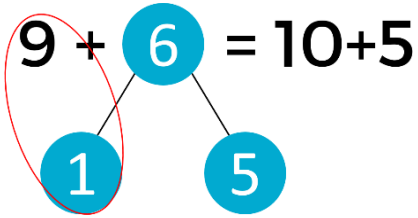
Identify number bond to 10 in order to regroup.

 
  </td>
<td> 
Draw number sentence starting with the largest number: 8 + 5 =.

Group 10 through identifying number bond to 10: 8+2=10.

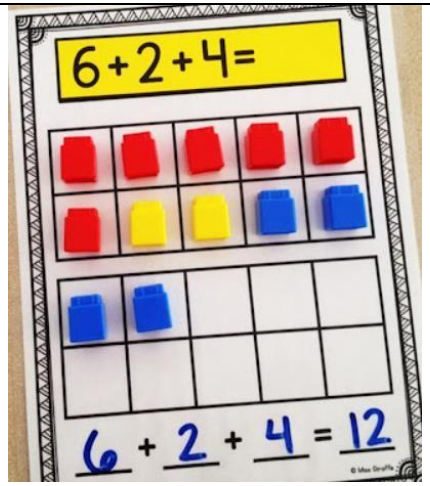
Add on remainder: 10 + 3 = 13

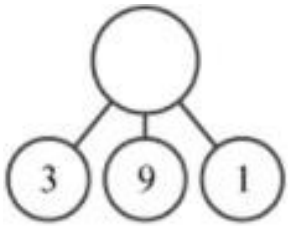
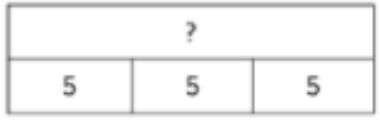
Use part, whole model to demonstrate regrouping.

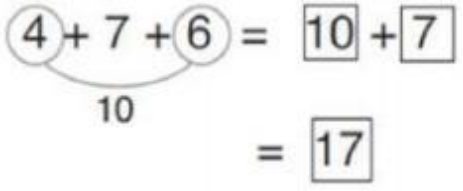
  </td>
<td>Number sentence:

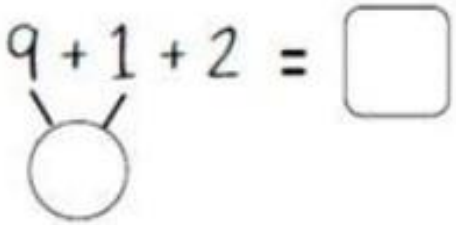
8 + 5 = 12

If I am at 8, how many more do I need to make 10?
I need 2 more (to make a group of 10).
How many more do I add on now?
I add 3 more on (because 2 + 3 = 5).</td>
</tr>
<tr>
<td>Adding 3 single digits through identifying number bonds.</td>
<td>Use practical apparatus to make the 3 numbers in the number sentence. Identify and combine the two numbers that form the number bond (a ten frame can be used to support children in identifying the number bond to 10). Add on the remainder.

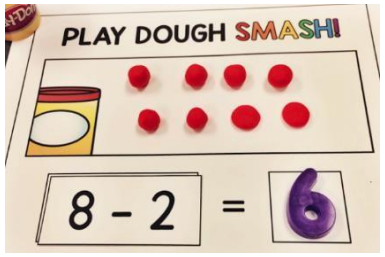
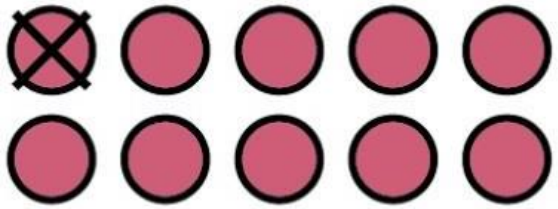
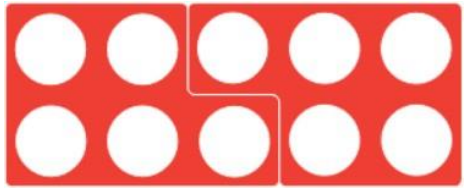
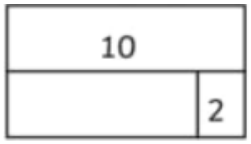
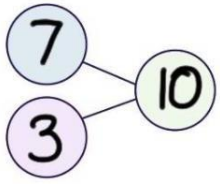
  </td>
<td>Identify the number bond through drawing, part, whole model or bar model. Combine the numbers that form the number bond and then add on the remainder:

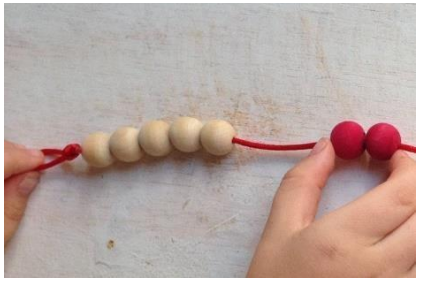
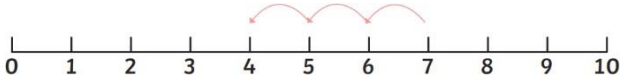
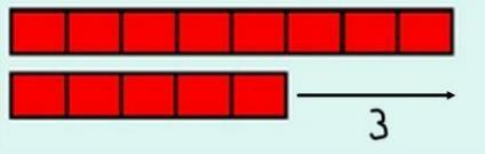
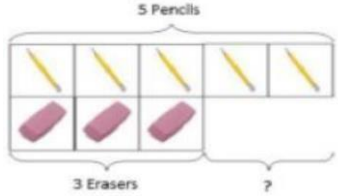
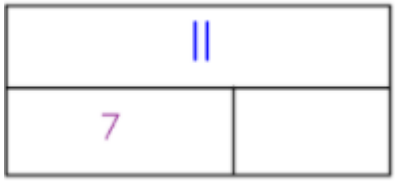
 
 
  </td>
<td>Number Sentence:

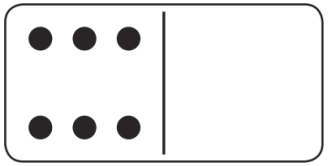
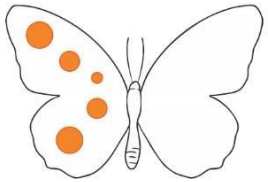
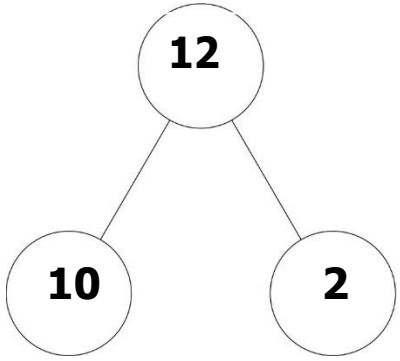
  </td>
</tr>
</table>

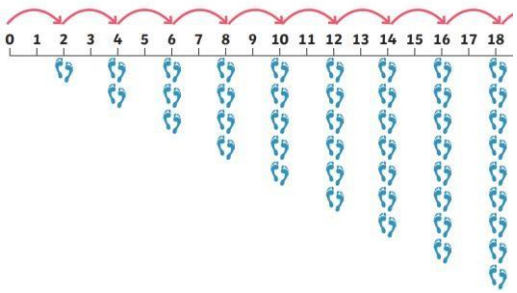
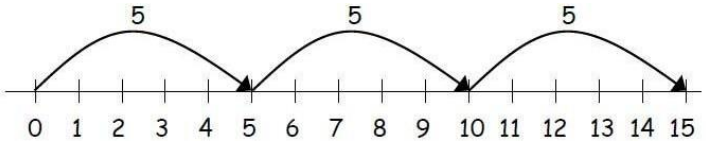
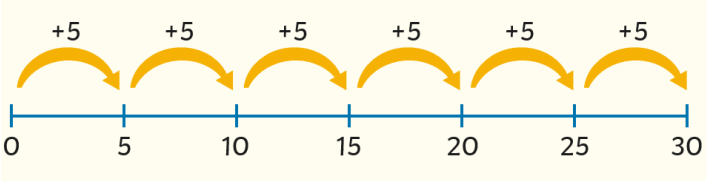
Year 1: Subtraction

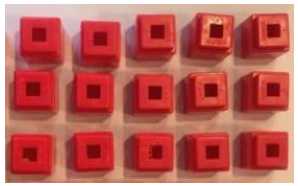
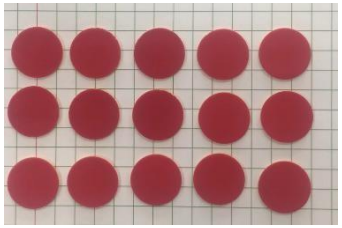
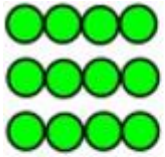
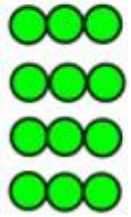
Strategy	Concrete	Pictorial	Abstract
Subtracting ones.	Use physical objects to show subtraction of ones.  	Draw total amount of objects. Cross out number being subtracted  10 – 1 =	Number sentence: **13 – 1 = 12** **7 = 9 – 2** **Equal symbol should be presented at the beginning and end of the number sentence to reinforce understanding of equals meaning same as/balance.**
Number bonds to 5 and 10.	Use counters, numicon or multi-link to make a whole (5 or 10) and take away a part. It is important to use this language.  **10 – 5 = 5**	Use pictures, part, whole model and bar model to take away from a group of 5/10.  **10 – 2 = 8**  **10 – 3 = 7**	Number sentences: **10 – 4 = 6** **5 = 10 – 5**

Counting backwards.	Use practical apparatus to subtract by making the largest number in the number sentence and counting backwards. Bead String: Move the beads along the string, counting backwards in ones.  **7 – 2 = 5** Counters/Cubes/Objects: Move the objects away, counting backwards in ones.	Use a number line or number track to count backwards, starting with the largest number and counting backwards in jumps of ones. 7 - 3 = 4  This can also be done using fingers/putting largest number in head and counting backwards.	Number sentence: 7 - 3 = 4 Mental Calculation: **13 – 4 = 9** **Put 13 in your head and count back 4. What number have you landed on?**
Finding the difference.	Use practical apparatus to show the difference between two numbers. Equipment such as multilink, which is equal in size and can be lined up exactly, demonstrates this concept.  **8 – 5 = 3**	Introduce bar models through drawing objects using a grid.  Use bar models to show finding the difference between two numbers.  	Number Sentence: **11 – 4 = 7** Number Stories: Hannah has 8 sweets. Jack has 3 sweets. Find the difference between the number of sweets. **8 – 3 = 5**

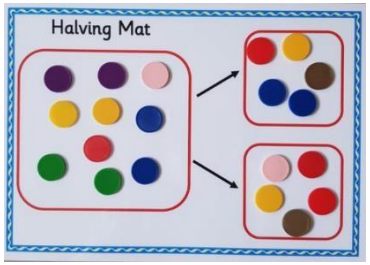
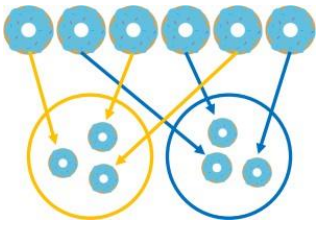
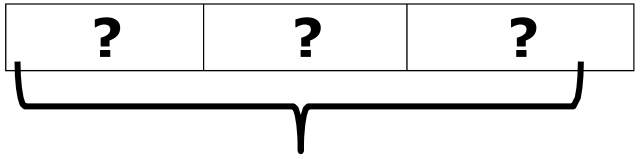
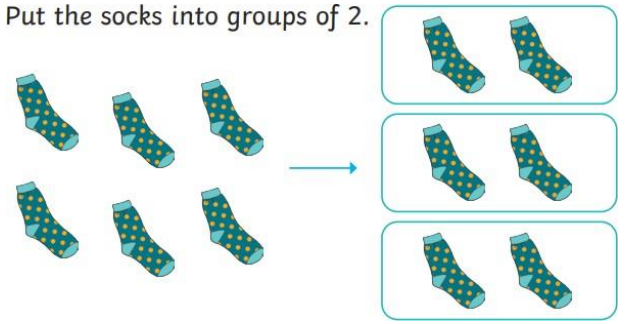
Year 1: Multiplication

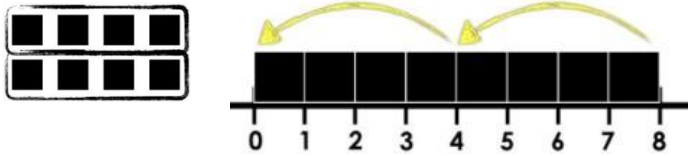
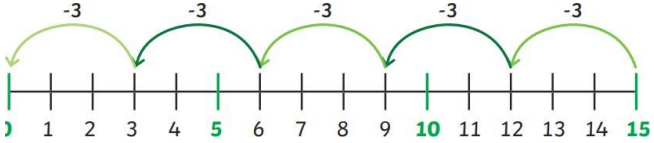
Strategy	Concrete	Pictorial	Abstract
Doubling.	Use physical apparatus/objects such as counters or multi-link to make one group/lot.  Double the amount to make two groups/lots and count how many there are in total. 	**One Digit** Draw the amount (one group/lot) and then draw two lots and count the total.   **Two Digit** Use part whole model to partition the number and double each part. Recombine to find the total.  **20 + 4**	**Number Sentence:** **4 + 4 = 8** **4 x 2 = 8** **2 x 4 = 8**

Counting in multiples.	Use practical apparatus/objects to count on in 2's. 	Count on using a number line or number track. 	**Number Sequence:** **2, 4, 6, 8, 10** **5, 10, ?, 20, ?**
Repeated Addition.	Use practical apparatus/objects to make groups for repeated addition.  **2 + 2 + 2 = 6**	**Repeated addition a number line:** **5 + 5 + 5 =**  Numbers or pictorial representations can be used beneath the number line to show intervals. **5 + 5 + 5 + 5 + 5 + 5 =** 	**Number Sentence:** **4 + 4 + 4 = 12** **4 x 3 = 12** **3 x 4 = 12**

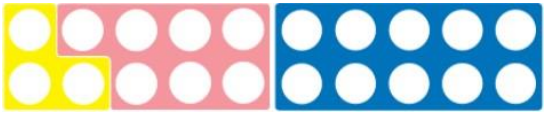
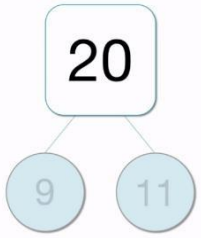
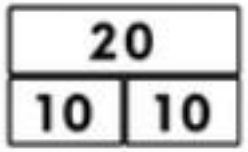
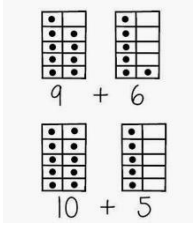
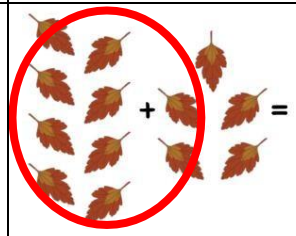
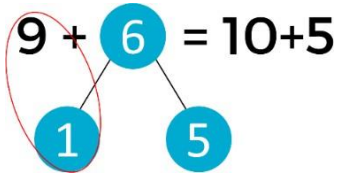
Use of arrays to show commutati -vity.	Create arrays using counters/cubes to show multiplication.  **3 x 5 =** 	Draw arrays to show multiplication.  3 x 4 = 12  4 x 3 =12 Arrays should be created in different rotations to demonstrate the commutative law.	**Number Sentence:** **4 x 3 = 12** **3 x 4 = 12**

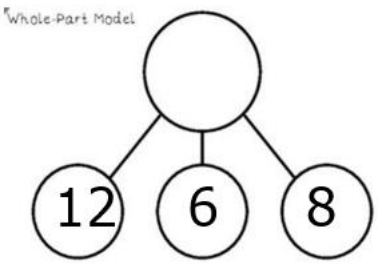
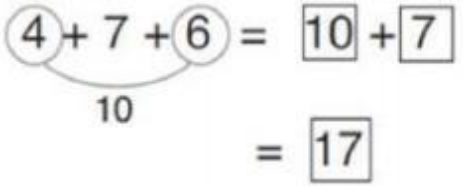
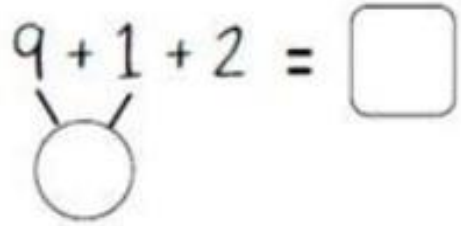
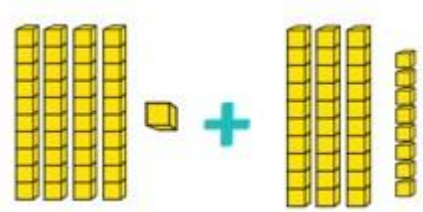
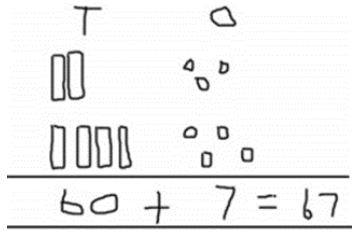
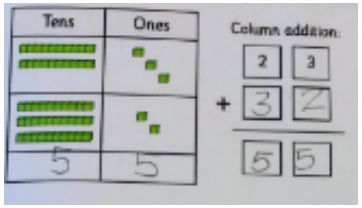
Year 1: Division

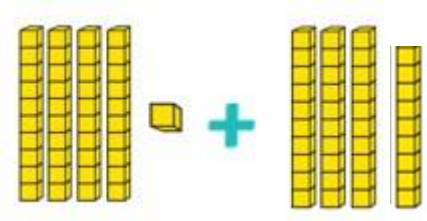
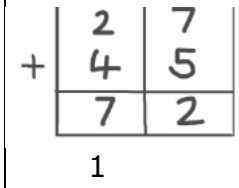
Strategy	Concrete	Pictorial	Abstract
Sharing (into equal groups)	Use physical apparatus/objects such as counters or multi-link share an amount into equal groups.  **Share the 9 cakes equally between the 3 bears.** Introduce halving as sharing into 2 equal groups.  **Half of 10**	Represent sharing into equal groups pictorially through drawing an amount being shared equally. Bar models can be used in this stage. **6 ÷ 2 = 3**   **6**	**Number Sentence:** **6 ÷ 3 = 2** **Half of 10 = 5**
Grouping	Use physical apparatus/objects such as counters or multi-link to put a given amount into equal groups.  	Represent grouping pictorially through drawing equal groups.  	**Number Sentence:** **6 ÷ 3 = 2**

Repeated subtraction (using a number line).	Use unifix cubes/bead strings to physically demonstrate how many times a smaller number goes into a larger number. Number lines can be used alongside bead strings/ unifix cubes. **8 ÷ 2 =**  **15 ÷ 3 =** 	Use repeated subtraction to demonstrate how many times a smaller number goes into a larger number. **15 ÷ 3 =**  	**Number Sentence:** **15 ÷ 3 =** The number of times you can take 3 from 15 is 5. **15 - 3 - 3 - 3 - 3 - 3 = 0** **15 ÷ 3 = 5**

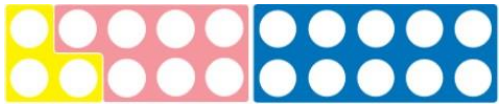
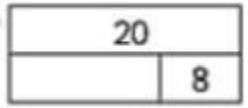
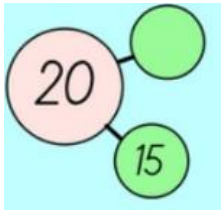
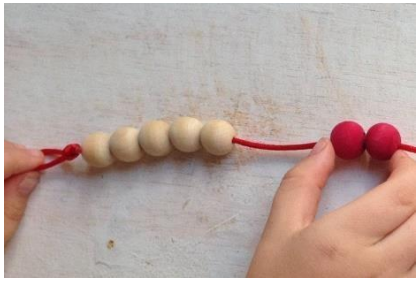
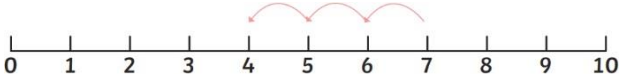
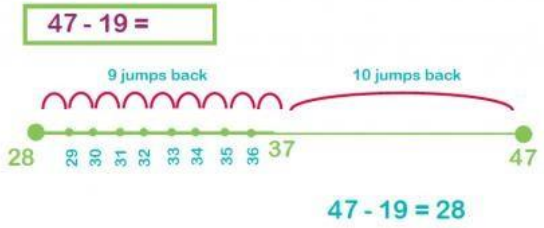
Year 2: Addition

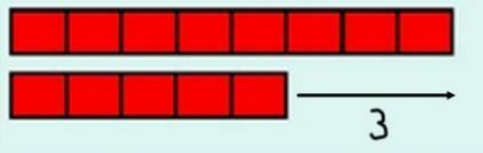
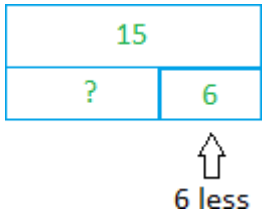
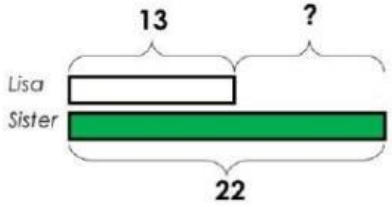
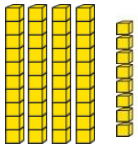
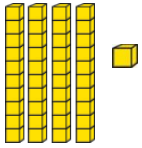
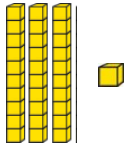
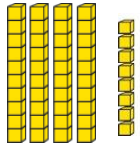
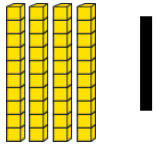
Strategy	Concrete	Pictorial	Abstract
Number bonds to 20.	Use counters (ten frame), numicon or multi-link to make/ combine two parts together to make a whole. It is important to use this language.  **17 + 3 = 20**	Use pictures to add two numbers together as a group of 20. **2 + 18 = 20**  Use part, part whole models/bar model to show number bonds to 20.   	Number Sentence: **8 + 12 = 20** **20 = 14 + 6** Equal symbol should be presented at the beginning and end of the number sentence to reinforce understanding of equals meaning same as/balance.
Regrouping to make groups of 10.	Use counters and a ten frame to show number sentence starting with the largest number. Identify number bond to 10 in order to regroup.  	 Draw number sentence starting with the largest number. Group 10 through identifying number bond to 10. Add on remainder. Use part, whole model to demonstrate regrouping.  	Number sentence: **8 + 5 = 12** **If I am at 8, how many more do I need to make 10?** **I need 2 more (to make a group of 10).** **How many more do I add on now?** **I add 3 more on (because 2 + 3 = 5).**

Adding 3 digits through identifying number bonds to 10 and 20.	Use practical apparatus to make the 3 numbers in the number sentence. Identify and combine the two numbers that form the number bond (a ten frame can be used to support children in identifying the number bond to 10). Add on the remainder.	Identify the number bond through drawing, part, whole model or bar model. Combine the numbers that form the number bond and then add on the remainder:   20 4 | 7 | 16	Number Sentence:  
Column addition without regrouping (two digit + two digit).	Use the dienes apparatus to physically makethe numbers, starting with the largest number (commutative law). Add ones, then add tens, then add hundreds working from right to left.  **41 + 38 =** NB: Place value knowledge must be secure in order to move onto this strategy.	Partition the number into tens and ones by drawing tens and ones in columns. Work from the right to the left, adding the ones first and then adding the tens.  	Number sentence using column addition without regrouping. T O 7 0 + 1 5 8 5

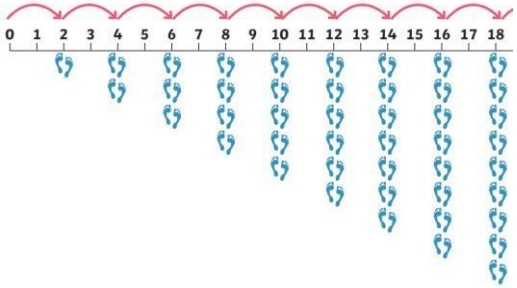
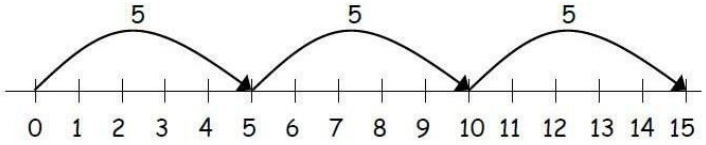
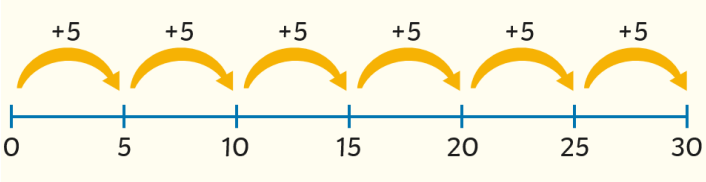
Column addition with regrouping (two digit + two digit).	Use dienes apparatus to physically make the numbers, starting with the largest number (commutative law). Add ones, then add tens, then add hundreds working from right to left.  **41 + 39 =** NB: Place value knowledge must be secure in order to move onto this strategy.	Partition the number into tens and ones drawing tens and ones in columns. Work from the right to the left, adding the ones first and then adding the tens.	Number sentence using column addition with regrouping. 

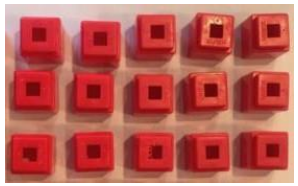
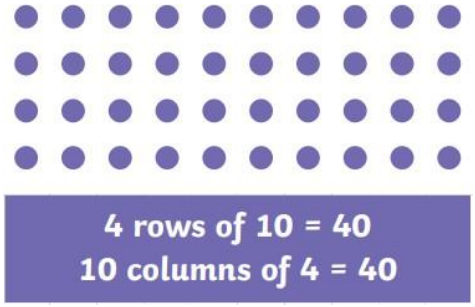
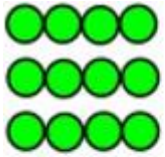
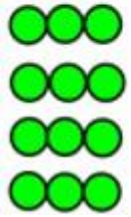
Year 2: Subtraction

Strategy	Concrete	Pictorial	Abstract
Number bonds to 20.	Use counters, numicon or multi-link to make a whole (5 or 10) and take away a part. It is important to use this language.  **20 - 3 = 17**	Use pictures, part, whole model and bar model to take away from a group of 20.  **20 – 8 = 12**  **20 – 15 = 5**	Number Sentence: **20 – 4 = 16** **15 = 20 – 5**
Counting backwards.	Use practical apparatus to subtract by making the largest number in the number sentence and counting backwards. Bead String: Move the beads along the string, counting backwards in ones.  **7 – 2 = 5** Counters/Cubes/Objects: Move the objects away, counting backwards in ones.	Use a number line or number track to count backwards, starting with the largest number and counting backwards in jumps of ones. 7 - 3 = 4  Use a number line or number track to count backwards in jumps of tens and jumps of one.  N.B: Place value knowledge must be secure (partitioning) in order to use this strategy.	Number sentence: **24 – 13 = 11** Mental Calculation: **24 – 3 = 21** **Put 24 in your head and count back 3. What number have you landed on?**

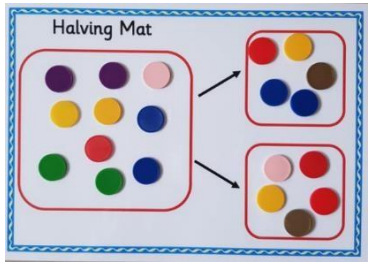
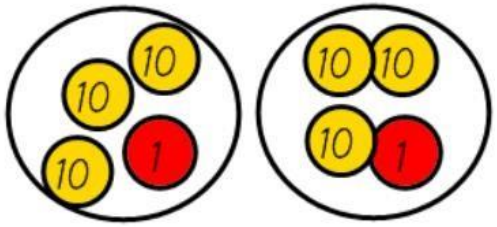
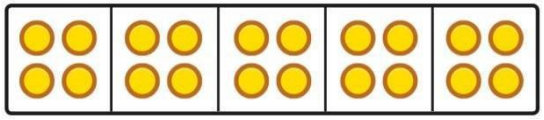
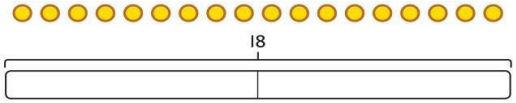
Finding the difference.	Use practical apparatus to show the difference between two numbers. Equipment such as multilink, which is equal in size and can be lined up exactly, demonstrates this concept.  **8 – 5 = 3**	Use bar models to show finding the difference between two numbers.  6 less *Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.* 	Number Sentence: **11 – 4 = 7** Number Stories: Hannah has 18 sweets. Jack has 13 sweets. Find the difference between the number of sweets. **18 – 13 = 5**
Expanded column subtraction without exchanging (two digits - two digits).	**48 – 17 =** Make the largest number using dienes apparatus.  Physically take away the ones.  Physically take away the tens. 	**48 – 17 =** Draw the largest number.  Cross out the ones.  Cross out the tens. 	Expanded column subtraction: TO - TO = 34 - 11 = 30 + 4 - 10 + 1 20 + 3 Partition the number into tens and ones. Work from the right to the left, subtracting the ones first and then subtracting the tens. Recombine the ones and the tens o find the answer.

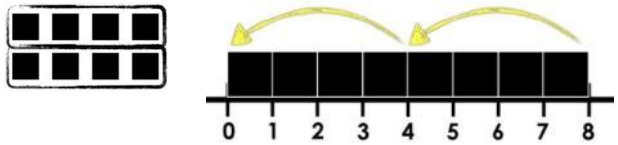
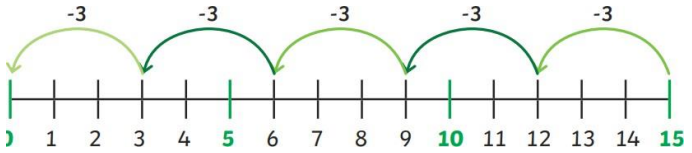
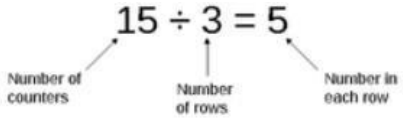
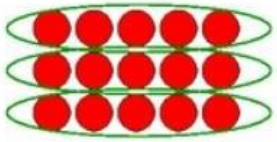
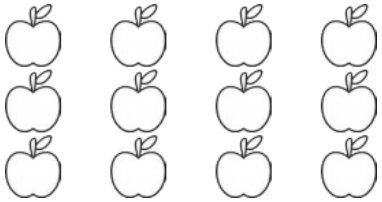
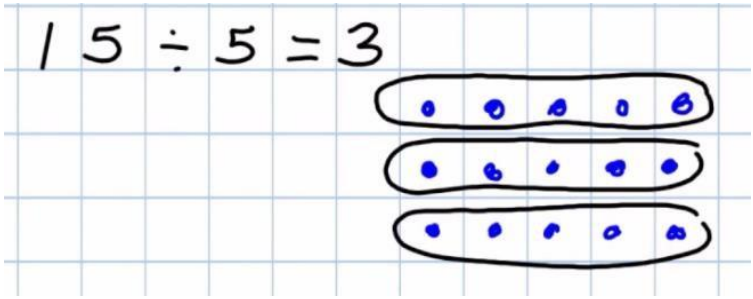
Year 2: Multiplication

Strategy	Concrete	Pictorial	Abstract
Counting in multiples.	Use practical apparatus/objects to count on in 2's. 	Count on using a number line or number track. 	**Number Sequence:** **2, 4, 6, 8, 10** **5, 10, ?, 20, ?**
Repeated Addition.	Use practical apparatus/objects to make groups for repeated addition.  **2 + 2 + 2 = 6**	**Repeated addition on a number line (skip-counting):** **5 + 5 + 5 =**  Numbers or pictorial representations can be used beneath the number line to show intervals. **5 + 5 + 5 + 5 + 5 + 5 =** 	**Number Sentence:** **4 + 4 + 4 = 12** **4 x 3 = 12** **3 x 4 = 12**

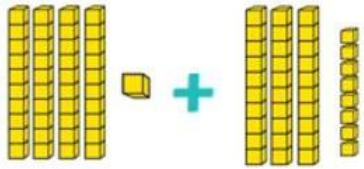
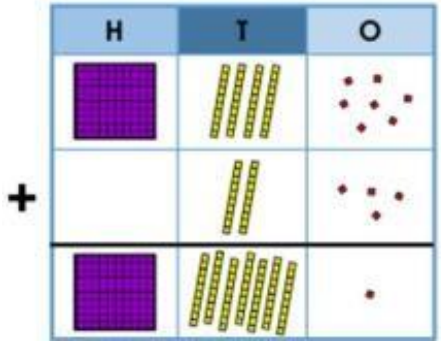
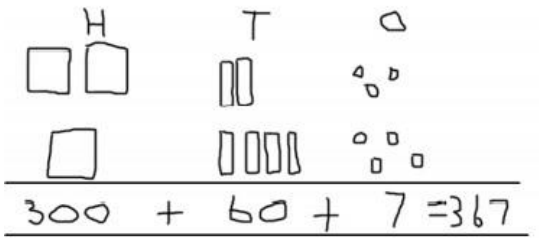
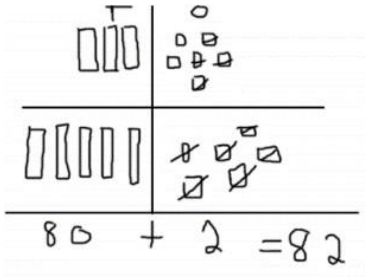
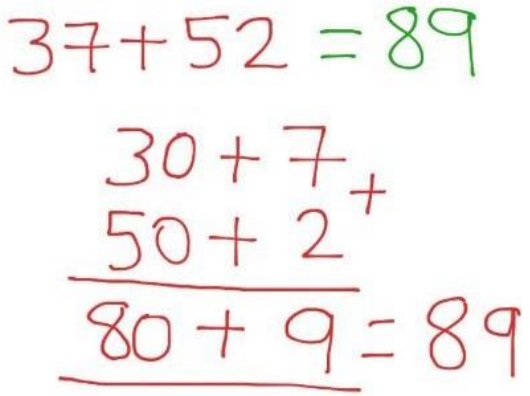
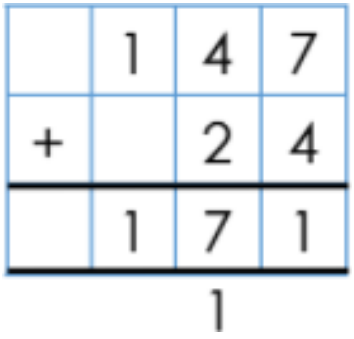
Use of arrays to show commutativity.	Create arrays using counters/cubes to show multiplication.  **3 x 5 =**  4 rows of 10 = 40 10 columns of 4 = 40 **4 x 10 =**	Draw arrays to show multiplication.  3 x 4 = 12  4 x 3 =12 Arrays should be created in different rotations to demonstrate the commutative law.	**Number Sentence:** **4 x 3 = 12** **3 x 4 = 12**

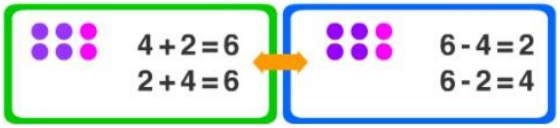
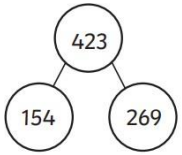
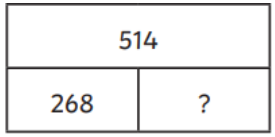
Year 2: Division

Strategy	Concrete	Pictorial	Abstract
Sharing	Use physical apparatus/objects such as counters or multi-link share an amount into equal groups.  Use place value counters to share larger quantities. **62 ÷ 2 = 31** Place value grids can also be used to support sharing larger quantities. One ten may need to be exchanged for ten ones. **42 ÷ 3 = 14** 	Represent the objects shared into equal parts using a bar model.  *20 shared into 5 equal parts. There are 4 in each part.*	Use a bar model to support understanding of the division.  *18 ÷ 2 = 9*

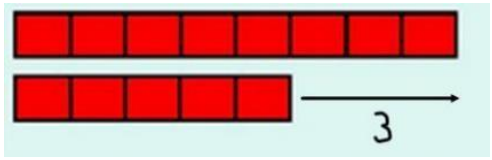
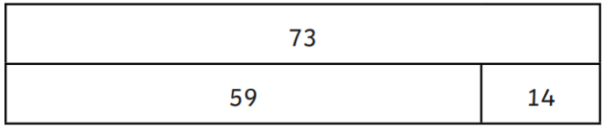
Repeated subtraction (using a number line).	Use unifix cubes/bead strings to physically demonstrate how many times a smaller number goes into a larger number. Number lines can be used alongside bead strings/ unifix cubes. **8 ÷ 2 =**  **15 ÷ 3 =** 	Use repeated subtraction to demonstrate how many times a smaller number goes into a larger number. **15 ÷ 3 =**  	**Number Sentence:** **15 ÷ 3 =** The number of times you can take 3 from 15 is 5. **15 - 3 - 3 - 3 - 3 - 3 = 0** **15 ÷ 3 = 5**
Arrays	Use physical apparatus/objects such as counters to create arrays.   **12 ÷ 3 = 4** **12 ÷ 4 = 3** 	Draw arrays to demonstrate division. **15 ÷ 5 = 3**  	**Number Sentence:** **12 ÷ 3 = 4** **There are 3 groups of 4.** **There are 4 groups of 3.**

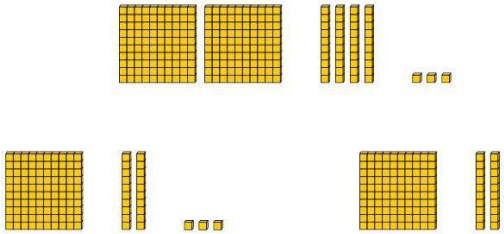
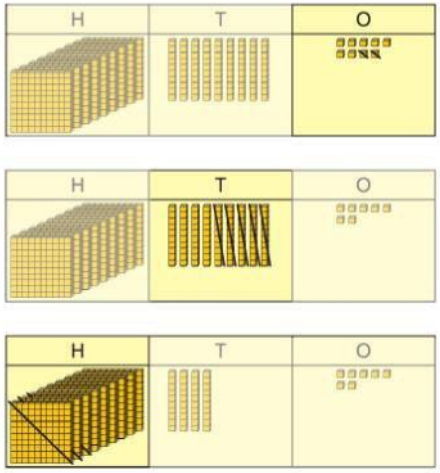
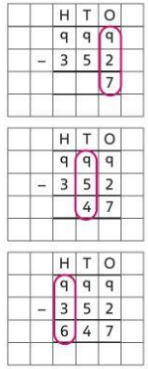
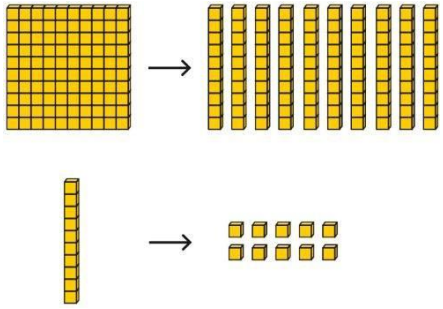
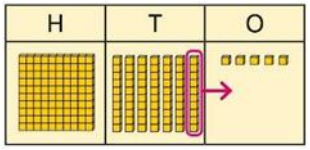
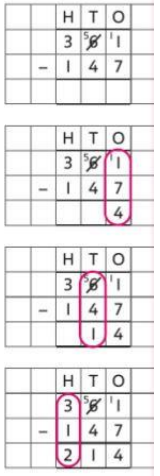
Year 3: Addition

Strategy	Concrete	Pictorial	Abstract
Expanded column addition <u>with and without</u> regrouping (**three digit + three digit).**	**Without regrouping:** Use dienes apparatus to physically make numbers, starting with the largest number (commutative law). Add ones, then add tens, then add hundreds working from right to left.  **41 + 38 =** **With regrouping:** Physically exchange ten ones for a ten and ten tens for a hundred when a ten or hundred boundary is met. **147 + 24 =** 	**Without regrouping:** **223 + 144 =**  Partition the number into tens and ones by drawing tens and ones in columns. Work from the right to the left, adding the ones first and then adding the tens. **With regrouping:** **37 + 46=**  There are 12 ones so exchange ten ones for a ten. Cross out ten ones and add the extra ten into the tens column. Add as normal by adding the ones first and then adding the tens.	Expanded column addition:  Partition the number into tens and ones. Work from the right to the left, adding the ones first and then adding the tens. Recombine the tens and the ones to find the answer.
Column addition (compact) <u>with and without</u> regrouping (**three digit + three digit).**			Column addition:  Work from the right to the left, beginning with the ones. When exchanges take place, they should be recorded beneath the calculation.

Using the inverse to check calculations.	Use practical apparatus such as counters, dienes apparatus, cubes etc. to form addition number sentences and then the related addition sentence using the commutative law and the related subtraction number sentences. **4 + 2 = 6**  Children need to understand the relationship between addition and subtraction as opposite operations.	Use pictorial models including bar models and part, whole models to show the inverse operation and the related number sentences. **154 + 269 = 423**  154 + 269 = 423 | 269 + 154 = 423 423 – 154 = 269 | 423 – 269 = 154 **268 + ? = 514**  **268 + ? = 514** **? + 268 = 514** **514 – 268 = ?** **514 - ? = 268**	Use formal methods for column addition and subtraction to demonstrate the inverse operation (including checking answers and calculating missing numbers). 

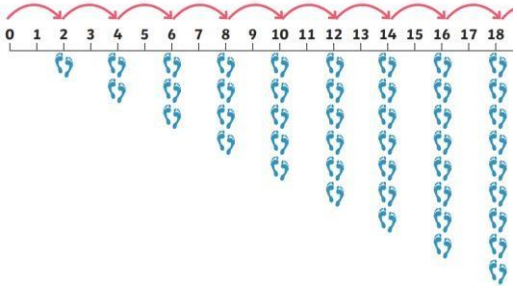
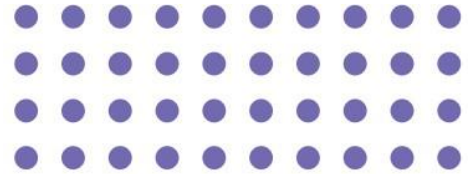
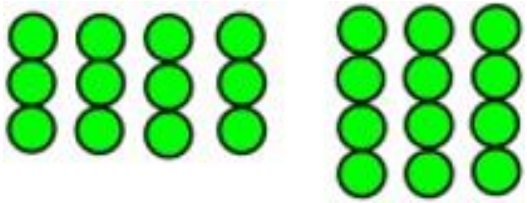
Year 3: Subtraction

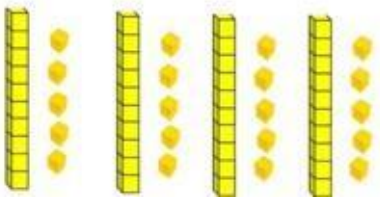
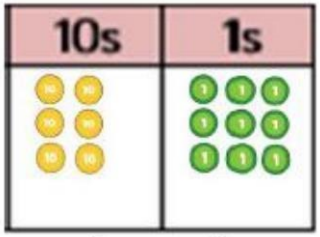
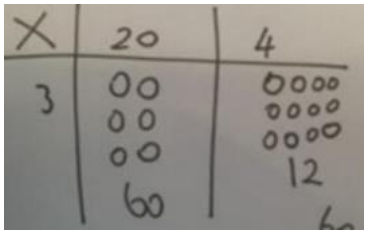
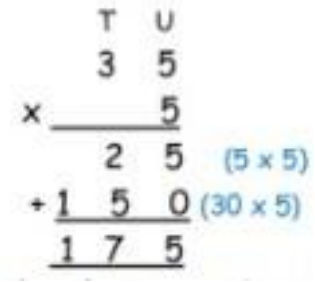
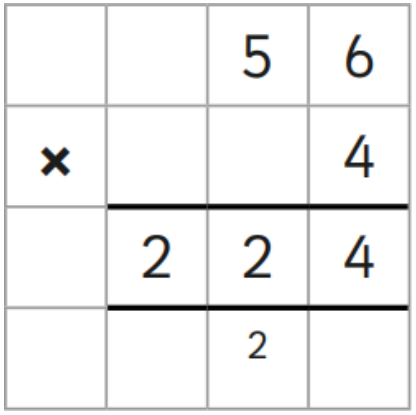
Strategy	Concrete	Pictorial	Abstract
Finding the difference.	Use practical apparatus to show the difference between two numbers. Equipment such as multilink, which is equal in size and can be lined up exactly, demonstrates this concept. 	Use bar models to show finding the difference between two numbers. What is the difference between 73 and 59? 	**Number Sentence:** What is the difference between 121 and 54? **121 – 54 =** **Number Stories:** Hannah has 108 sweets. Jack has 113 sweets. Find the difference between the number of sweets. **113 – 108 =**

Compact column subtraction <u>with and</u> without exchanging (**three digit - three digit/two digit).**	Use place value equipment to explore the effect of splitting a whole into two parts, andunderstand the link with taking away. 	Represent the calculation on a place valuegrid. 	Use column subtraction to calculate accurately and efficiently. 
	Use place value equipment to enact the exchange of 1 hundred for 10 tens, and 1 	*175 − 38 = ?* *I need to subtract 8 ones, so I will exchangea ten for 10 ones.* 	Use column subtraction to work accurately and efficiently.  If the subtraction is a 3-digit number subtract a 2-digit number, children should understand how the recording relates to the place value, and so how to line up the digits correctly.

			Children should also understand how to exchange in calculations where there is a zero in the 10s column.

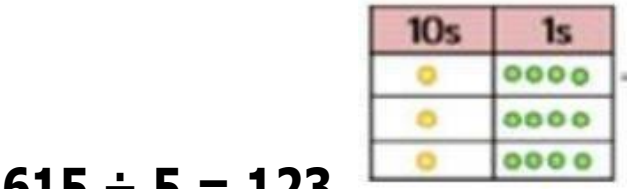
Year 3: Multiplication

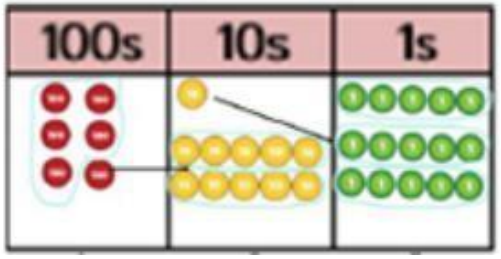
Strategy	Concrete	Pictorial	Abstract
Counting in multiples.	Use practical apparatus/objects to count on in 2's. 	Count on using a number line or number track. 	**Number Sequence:** **2, 4, 6, 8, 10** **5, 10, ?, 20, ?**
Use of arrays to show commutativity.	Create arrays using counters/cubes to show multiplication.  **4 x 10 =** 4 rows of 10 = 40 10 columns of 4 = 40	Draw arrays to show multiplication.  Arrays should be created in different rotations to demonstrate the commutative law.	**Number Sentence:** **4 x 3 = 12** **3 x 4 = 12**

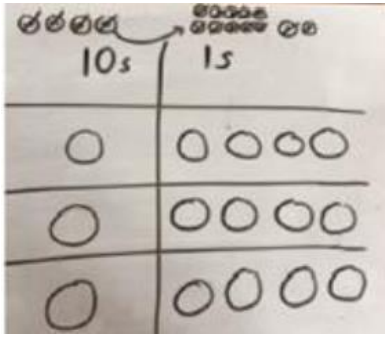
Expanded method of short multiplication (two digit by one digit)	Use dienes apparatus to make groups. Combine units and tens. Add together to find the total.  **4 x 15 =** **4 x 10 = 40** **4 x 5 = 20** **40 + 20 = 60** Use counters to represent value of digits to multiply in a place value grid. Recombine tens and ones.  **23 x 3 =** **20 x 3 = 60** **3 x 3 = 9** **60 + 9 = 69**	Draw dienes apparatus or counters to represent place value of digits in columns. **24 x 3 =** **60 + 12 = 72**  	**Expanded Method of Short Multiplication:**  Multiply from the right to the left (ones and then tens). When exchanges take place, they should be recorded beneath the calculation. **Short Multiplication:** 

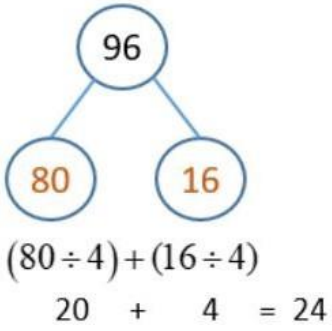
Year 3: Division

Strategy	Concrete	Pictorial	Abstract
Short Division Y3: Use short division to divide two digit numbers by one digit numbers. Y4: Use short division to divide three digit numbers by one digit numbers	Place value grids can also be used to support sharing larger quantities. One ten may need to be exchanged for ten ones. **42 ÷ 3 = 14** **615 ÷ 5 = 123** Make 615 with place value counters. How many groups of 5 hundreds can you make with 6 hundred counters? Exchange 1 hundred for 10 tens. How many groups of 5 tens can you make with 11 counters? Exchange 1 ten for 10 ones. How many groups of 5 ones can you make with 15 ones?	**Place Value Grid:** **42 ÷ 3 = 14** Draw total amount (4 tens and 2 ones). Divide into 3 equal groups. Cross out counters as they are shared. Where a ten cannot be shared equally, exchange for ten ones so that it can be shared equally. **Part-Whole Model:** **96 ÷ 4 = 24** $(80 \div 4) + (16 \div 4)$ 20 + 4 = 24	**Number Sentence:** Without carrying: How many 4's in 8 (tens)? How many 4's in 4 (ones)? With carrying: How many 3's in 4(tens)? Exchange the remaining ten. How many 3's in 15?





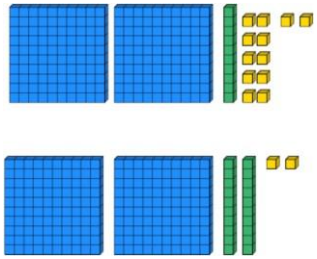
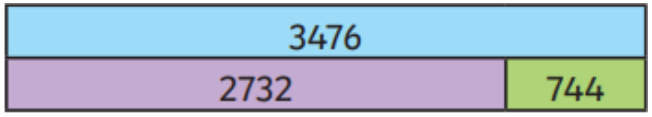


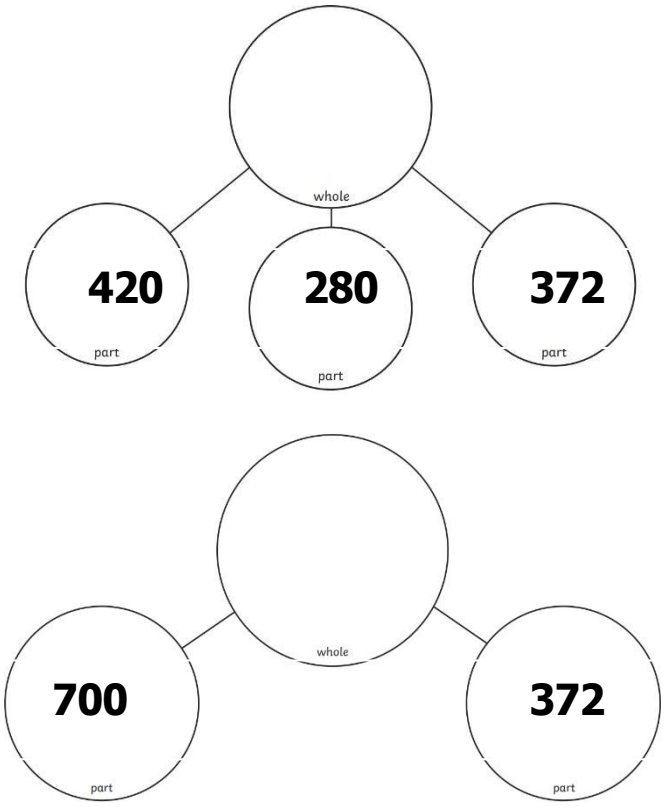


	2	1
4	8	4

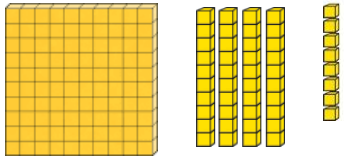
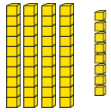
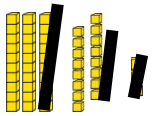
	1	5
3	4	$^{1}5$

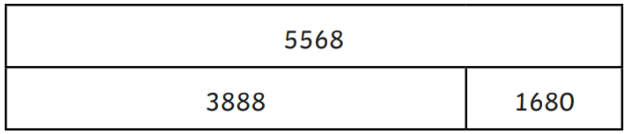
Year 4: Addition

Strategy	Concrete	Pictorial	Abstract
Column addition (compact) <u>with and without</u> regrouping /exchanging (**four digit + four digit).**	**Without regrouping:** Use dienes apparatus to physically add thousands, hundreds, tens and ones. **With regrouping:** **119 + 103 = 222** Physically exchange ten ones for a ten, ten tens for a hundred and ten hundreds for a thousand. 	**Without regrouping:** Draw dienes apparatus and add ones first, then add tens, then add hundreds and finally add thousands. **With regrouping:** Draw dienes apparatus and **to add from the right to the left, beginning with the ones as with compact column addition.** When exchanging, cross out and regroup e.g. Cross out ten ones and add the extra ten into the tens column.	**Without regrouping/exchanging:** 5162 +3427 8589 **With one regroup/ exchange:** 5162 +3497 8659 1 **With multiple regroup/exchanges:** 5864 +3497 9361 11 1 Work from the right to the left, beginning with the ones. When exchanges take place, they should be recorded beneath the calculation.
Using the inverse to check calculations.	Use practical apparatus such as counters, dienes apparatus, cubes etc. to form addition number sentences and then the related addition sentence using the commutative law and the related subtraction number sentences.	Use pictorial models including bar models and part, whole models to show the inverse operation and the related number sentences. **3476 – 744 = 2732**  3476 – 744 = 2732 can be checked using 2732 + 744 = 3476	Use formal methods for column addition and subtraction to demonstrate the inverse operation (including checking answers and calculating missing numbers). **5162 + 3497 = 8659** **3497 + 5162 = 8659** **8659 – 3497 = 5162** **8659 – 5162 = 3497** 5162 +3497 8659 1

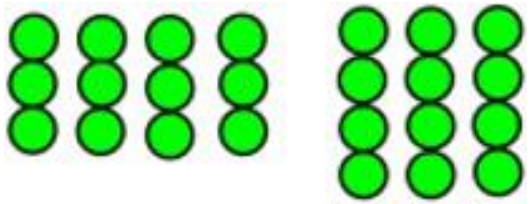
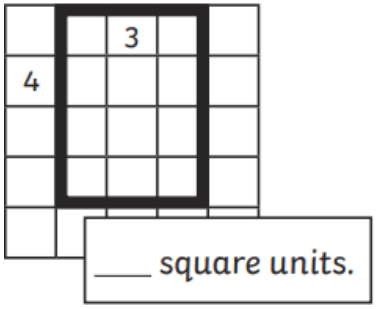
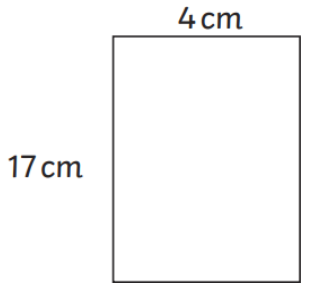
Changing the order of numbers through identifying number bonds to check calculations.	Practical apparatus such as counters, dienes apparatus, cubes etc. can be used to form addition number sentences and physically manipulated to demonstrate known number facts e.g. 60 + 40 = 100 and the commutative law (numbers can be added in any order to get the total sum).	Use pictoral models including bar models and part, whole models to demonstrate known number bonds. **420 + 372 + 280 =** 	Identify useful number bonds in order to rewrite a number sentence and recalculate to check answer. **420 + 372 + 280 =** **Change to 420 + 280 + 372 as 420 + 280 = 700** **(because 42 + 28 = 70 (number bond))** **420 + 280 + 372 = 700 + 372 = 1072**

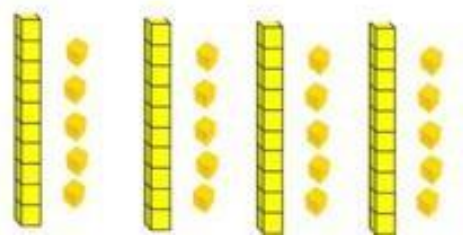
Year 4: Subtraction

Strategy	Concrete	Pictorial	Abstract
Compact column subtraction <u>with and without</u> exchanging (**up to four digits).**	**Without exchanging:** **148 – 17 =** Physically take away the ones, then the tens and then the hundreds.  **With exchanging:** **32 – 7 =** Make the largest number using dienes apparatus. Physically take away the ones, then the tens and finally the hundreds. If there are not enough ones, exchange one ten for ten units. If there are not enough tens, exchange one hundred for ten tens.	**Without exchanging:** Draw the largest numbers. Cross out the ones being taken away, followed by the tens and then the hundreds. **With exchanging:** **47 – 19 =** Draw the largest numbers.  If there are not enough ones, exchange one ten for ten units. If there are not enough tens, exchange one hundred for ten tens. Cross out the ones being taken away followed by the tens and the units. 	**Without exchanging:** 5789 - 3421 2368 **With one exchange:** 6 1 5749 - 3471 2278 **With multiple exchange:** 6 13 1 5742 - 3476 2266

Finding the difference.	Use practical apparatus to show the difference between two numbers. Equipment such as multilink, which is equal in size and can be lined up exactly, demonstrates this concept.	Use bar models to show finding the difference between two numbers. What is the difference between 5568 and 3888? 	**Number Sentence:** What is the difference between 1216 and 504? **1216 – 504 =**

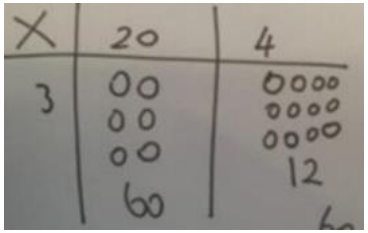
Year 4: Multiplication

Strategy	Concrete	Pictorial	Abstract
Use of arrays to show commutativity)	Create arrays using counters/cubes to show multiplication. **4 x 10 =** 	Draw arrays to show multiplication. Arrays should be created in different rotations to demonstrate the commutative law.  **Calculating area:** Use squares to create arrays when calculating the area of rectangles. 	**Number Sentence:** **4 x 3 = 12** **3 x 4 = 12** **Calculating area:** Calculate the area of this rectangle. **4 x 17 =** 

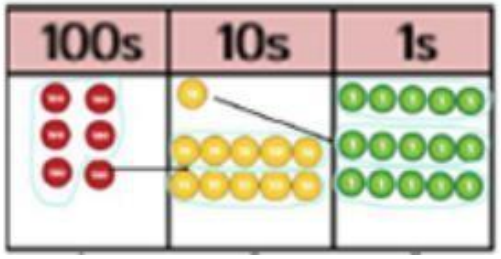
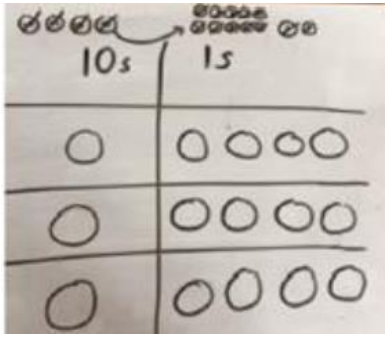
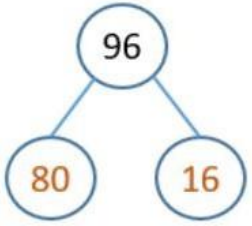
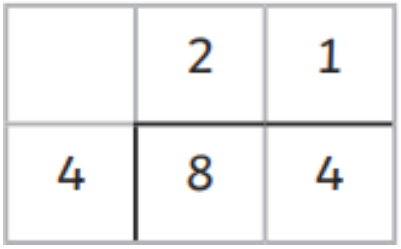
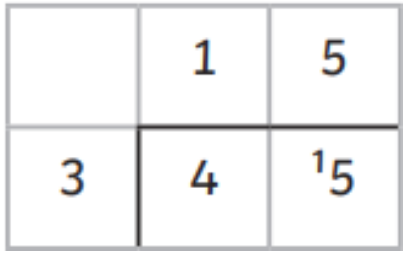
<table>
<tr>
<td>Expanded method of short multiplication (three digit by one digit)</td>
<td>Use dienes apparatus to make groups.
Combine units and tens.
Add together to find the total.
 
4 x 15 =
4 x 10 = 40
4 x 5 = 20
40 + 20 = 60

Use counters to represent value of digits to multiply in a place value grid.
Recombine tens and ones.
 

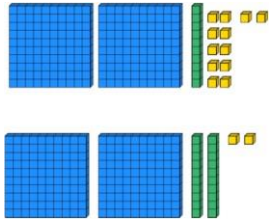
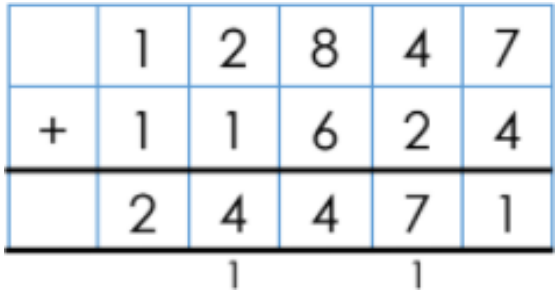
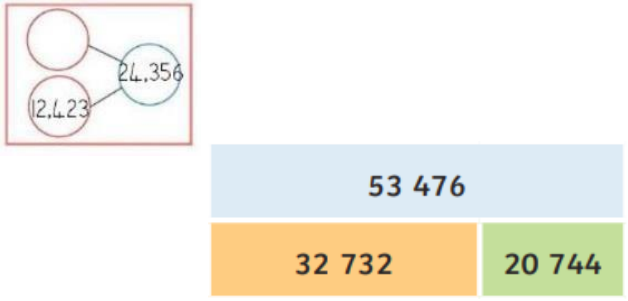
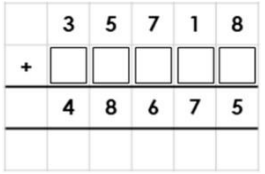
23 x 3 =
20 x 3 = 60
3 x 3 = 9
60 + 9 = 69</td>
<td>Draw dienes apparatus or counters to represent place value of digits in columns.
24 x 3 =

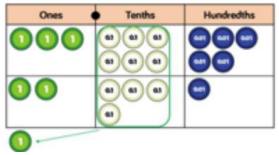
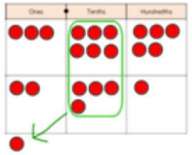
60 + 12 = 72
 
</td>
<td>Expanded Method of Short Multiplication:
<table>
<tr><th>Th</th><th>H</th><th>T</th><th>O</th><th></th></tr>
<tr><td></td><td>5</td><td>4</td><td>3</td><td></td></tr>
<tr><td>×</td><td></td><td></td><td>4</td><td></td></tr>
<tr><td></td><td></td><td>1</td><td>2</td><td>(4 × 3)</td></tr>
<tr><td></td><td>1</td><td>6</td><td>0</td><td>(4 × 40)</td></tr>
<tr><td>2</td><td>0</td><td>0</td><td>0</td><td>(4 × 500)</td></tr>
<tr><td>2</td><td>1</td><td>7</td><td>2</td><td></td></tr>
</table>
Multiply from the right to the left (ones, tens and then hundreds). When exchanges take place, they should be recorded beneath the calculation.</td>
</tr>
<tr>
<td>Short multiplication (three digit by one digit).</td>
<td>Short Multiplication:
<table>
<tr><th>Th</th><th>H</th><th>T</th><th>O</th></tr>
<tr><td></td><td>5</td><td>4</td><td>3</td></tr>
<tr><td>×</td><td></td><td></td><td>4</td></tr>
<tr><td>2</td><td>1</td><td>7</td><td>2</td></tr>
<tr><td></td><td>1</td><td>1</td><td></td></tr>
</table>
Multiply from the right to the left (ones, tens and then hundreds). When exchanges take place, they should be recorded beneath the calculation.</td>
</tr>
</table>

Year 4: Division

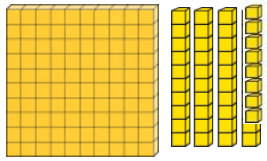
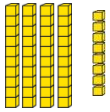
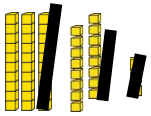
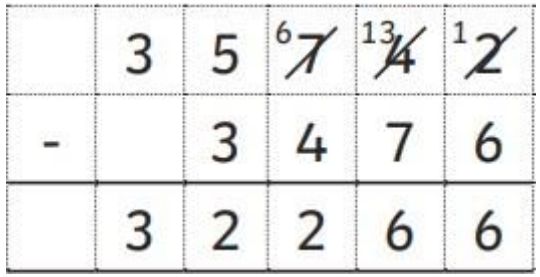
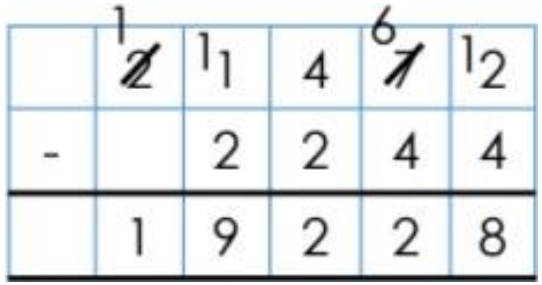
Strategy	Concrete	Pictorial	Abstract
Short Division Y3: Use short division to divide two digit numbers by one digit numbers. Y4: Use short division to divide three digit numbers by one digit numbers	Place value grids can also be used to support sharing larger quantities. One ten may need to be exchanged for ten ones. **42 ÷ 3 = 14**  **615 ÷ 5 = 123**  Make 615 with place value counters. How many groups of 5 hundreds can you make with 6 hundred counters? Exchange 1 hundred for 10 tens. How many groups of 5 tens can you make with 11 counters? Exchange 1 ten for 10 ones. How many groups of 5 ones can you make with 15 ones?	**Place Value Grid:** **42 ÷ 3 = 14** Draw total amount (4 tens and 2 ones). Divide into 3 equal groups. Cross out counters as they are shared. Where a ten cannot be shared equally, exchange for ten ones so that it can be shared equally.  **Part-Whole Model:** **96 ÷ 4 = 24**  $(80 \div 4) + (16 \div 4)$ 20 + 4 = 24	**Number Sentence:** Without carrying:  How many 4's in 8 (tens)? How many 4's in 4 (ones)? With carrying:  How many 3's in 4(tens)? Exchange the remaining ten. How many 3's in 15?

Year 5: Addition

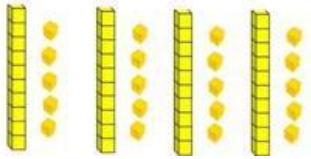
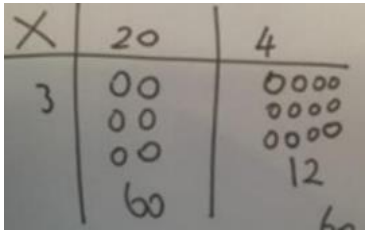
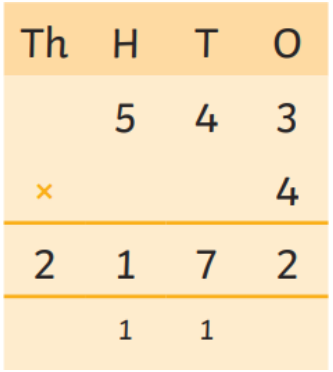
Strategy	Concrete	Pictorial	Abstract
Column addition (compact) <u>with and without</u> regrouping/ exchanging **(addition up to and including five digit numbers).**	**Without regrouping:** Use dienes apparatus to physically add thousands, hundreds, tens and ones. **With regrouping:** **119 + 103 = 222** Physically exchange ten ones for a ten, ten tens for a hundred and ten hundreds for a thousand. 	**Without regrouping:** Draw dienes apparatus and add ones first, then add tens, then add hundreds and finally add thousands. **With regrouping:** Draw dienes apparatus and **to add from the right to the left, beginning with the ones as with compact column addition.** When exchanging, cross out and regroup e.g. Cross out ten ones and add the extra ten into the tens column.	**Without regrouping, one regroup/exchange and multiple regrouping/exchanges:**  Work from the right to the left, beginning with the ones. When exchanges take place, they should be recorded beneath the calculation.
Using the inverse to check calculations and identify missing numbers.	Use practical apparatus such as counters, dienes apparatus, cubes etc. to form addition number sentences and then the related addition sentence using the commutative law and the related subtraction number sentences.	Use pictorial models including bar models and part, whole models to show the inverse operation and the related number sentences. E.g. **53,476 - ? = 20,744** **24,356 = ? + 12,423** 	Use formal methods for column addition and subtraction to demonstrate understanding of the inverse operation. **35,718 + ? = 48,675**  **75,351 – 40,428 =** The difference between 34,623 and 75,351 is 40,728. Use the inverse to check this statement.

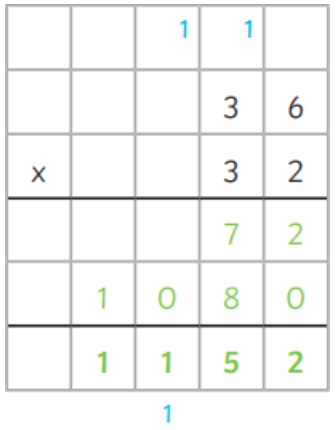
Practise adding and subtracting decimals, including a mix of whole numbers and decimals, decimals with different numbers of decimal places, and complements of 1 (for example, 0.83 + 0.17 = 1).	Use practical apparatus such as counters, dienes apparatus, cubes to represent adding and subtracting decimals.  	Use pictorial models including bar models and part, whole models to show the inverse operation and the related number sentences.	Use formal methods for column addition and subtraction to demonstrate understanding of the inverse operation. 

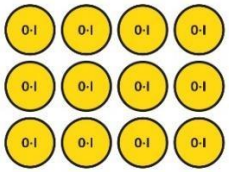
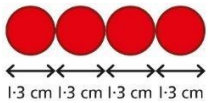
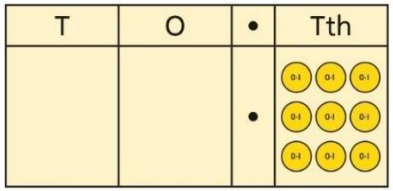
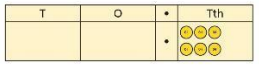
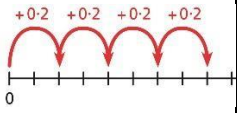
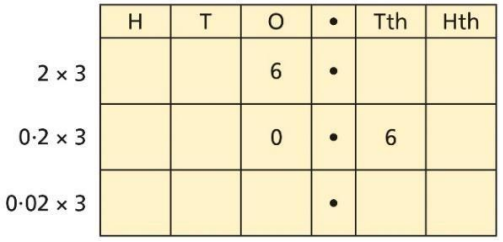
Year 5: Subtraction

Strategy	Concrete	Pictorial	Abstract
Compact column subtraction <u>with and without</u> exchanging.	**Without exchanging:** **148 – 17 =** Physically take away the ones, then the tens and then the hundreds.  **With exchanging:** **32 – 7 =** Make the largest number using dienes apparatus. Physically take away the ones, then the tens and finally the hundreds. If there are not enough ones, exchange one ten for ten units. If there are not enough tens, exchange one hundred for ten tens.	**Without exchanging:** Draw the largest numbers. Cross out the ones being taken away, followed by the tens and then the hundreds. **With exchanging:** **47 – 19 =** Draw the largest numbers.  If there are not enough ones, exchange one ten for ten units. If there are not enough tens, exchange one hundred for ten tens. Cross out the ones being taken away followed by the tens and the units. 	**With and without exchanging:**  
Finding the difference.	Use practical apparatus to show the difference between two numbers. Equipment such as multilink, which is equal in size and can be lined up exactly, demonstrates this concept.	Use bar models to show finding the difference between two numbers. What is the difference between 25, 567 and 1,875?  	**Number Sentence:** What is the difference between 102,616 and 14,504? **102,616 – 14,504 =**

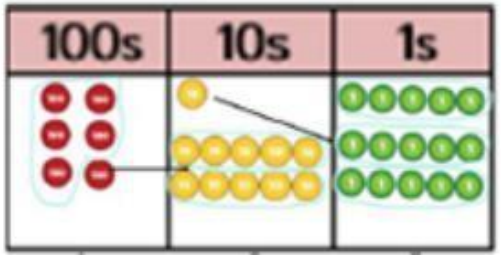
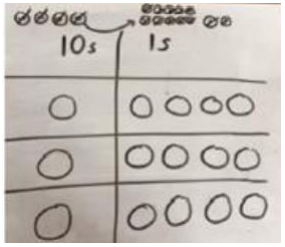
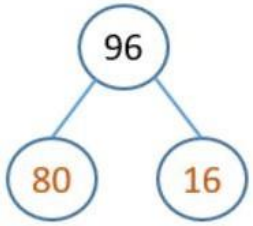
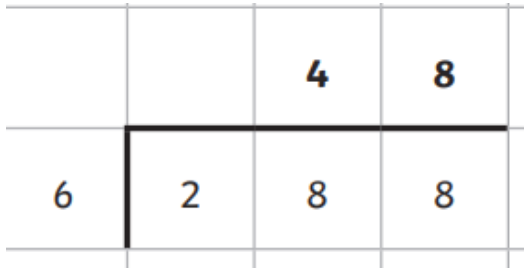
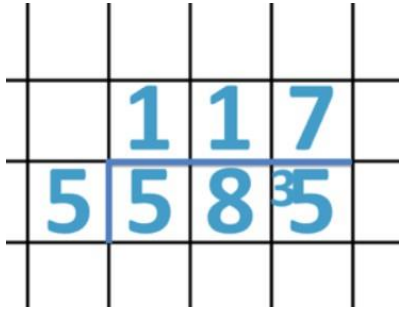
Year 5: Multiplication

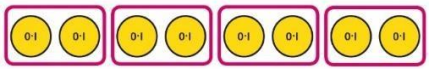
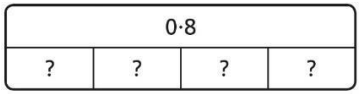
Strategy	Concrete	Pictorial	Abstract
Short Multiplication	Use dienes apparatus to make groups. Combine units and tens. Add together to find the total.  **4 x 15 =** **4 x 10 = 40** **4 x 5 = 20** **40 + 20 = 60** Use counters to represent value of digits to multiply in a place value grid. Recombine tens and ones.  **23 x 3 =** **20 x 3 = 60** **3 x 3 = 9** **60 + 9 = 69**	Draw dienes apparatus or counters to represent place value of digits in columns. **24 x 3 =** **60 + 12 = 72**  	**Short Multiplication:**  Multiply from the right to the left (ones, tens and then hundreds). When exchanges take place, they should be recorded beneath the calculation.

Long Multiplication	***NB: CPA understanding must be in place for short multiplication in order to progress to long multiplication.***	**Long Multiplication**  Begin long multiplication with the ones. Multiply 2 x 6. Write the answer down correctly, recording any exchanges above the calculation in the correct column. 6 x 2 = 12 so place the 2 in the ones column and carry the ten above the calculation. Multiply 2 x 3 (tens). Write the answer down correctly, recording any exchanges above the correct column. 3 x 2 = 6 + 1 ten = 7 tens. Place a zero in the row below in the ones column as the next step requires multiplying by 10. Multiply 3 (tens) x 6. Write the answer down correctly recording any exchanges above the calculation in the correct column. 3 (tens) x 6 = 18. Place the 8 in the tens column and carry the 1 (hundred) into the hundreds column. Multiply 3 (tens) by 3 (tens). Write the answer down correctly recording any exchanges above the calculation in the correct column. 3 (tens) x 3 (tens) = 9. Add 9 to the 1 in the hundreds column and record in the thousands column. Add 1082 + 72. Record any exchanges beneath the calculation.

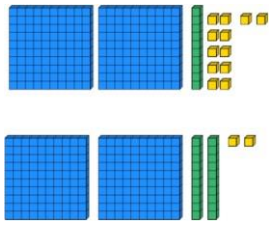
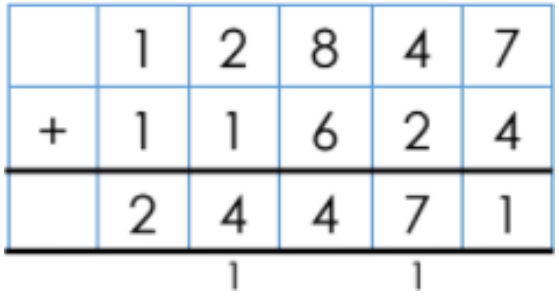
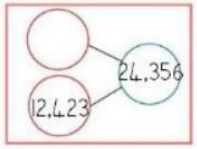
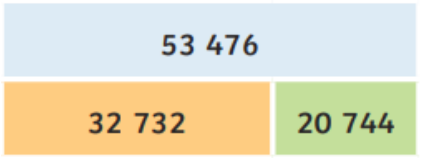
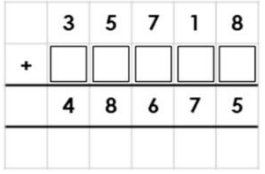
Multiplying Decimals	Explore decimal multiplications using placevalue equipment and in the context of measures.  *3 groups of 4 tenths is 12 tenths.* *4 groups of 3 tenths is 12 tenths.*  *4 × 1 cm = 4 cm* *4 × 0·3 cm = 1.2 cm* *4 × 1·3 = 4 + 1·2 = 5·2 cm*	Represent calculations on a place valuegrid. 3 × 3 = 9 3 × 0·3 = 0·9  Understand the link between multiplyingdecimals and repeated addition.   	Use known facts to multiply decimals. *4 × 3 = 12* *4 × 0·3 = 1·2* *4 × 0·03 = 0·12* *20 × 5 = 100* *20 × 0·5 = 10* *20 × 0·05 = 1* Find families of facts from a known multiplication. *I know that 18 × 4 = 72. This can help me* *work out:* *1·8 × 4 = ?* *18 × 0·4 = ?* *180 × 0·4 = ?* *18 × 0·04 = ?* Use a place value grid to understand theeffects of multiplying decimals. 

Year 5: Division

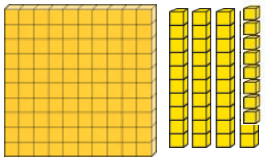
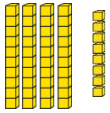
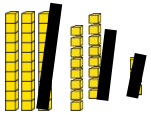
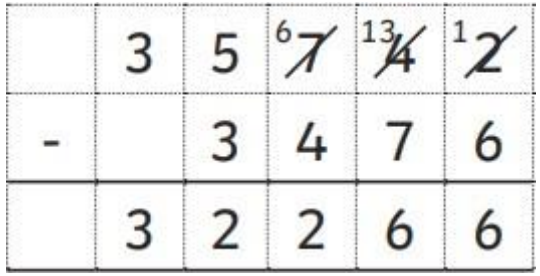
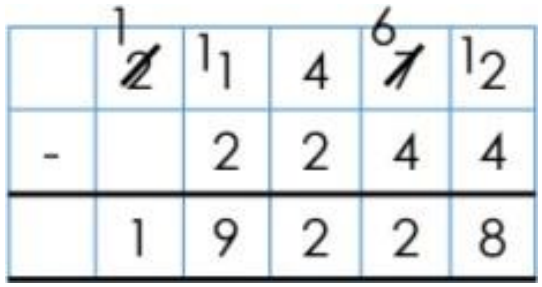
Strategy	Concrete	Pictorial	Abstract
Short Division: Use short division to divide three and four digit numbers by one digit numbers.	**615 ÷ 5 = 123**  Make 615 with place value counters. How many groups of 5 hundreds can you make with 6 hundred counters? Exchange 1 hundred for 10 tens. How many groups of 5 tens can you make with 11 counters? Exchange 1 ten for 10 ones. How many groups of 5 ones can you make with 15 ones?	**Place Value Grid/Part-Whole Model** **42 ÷ 3 = 14**  **96 ÷ 4 = 24**  $(80 \div 4) + (16 \div 4)$ 20 + 4 = 24	**Number Sentence:** Without carrying:  With carrying:  How many 5's in 5 (hundreds)? How many 5's in 8 (tens)? Exchange the remaining 3 tens. How many 5's in 35?

Dividing Decimals	Use place value equipment to explore division of decimals.  *8 tenths divided into 4 groups. 2 tenths in each group.*	Use a bar model to represent divisions.  0·8 ? ? ? ? $4 \times 2 = 8$ $8 \div 4 = 2$ So, $4 \times 0{\cdot}2 = 0{\cdot}8$ $0{\cdot}8 \div 4 = 0{\cdot}2$	Use short division to divide decimals with upto 2 decimal places. $8 \overline{)4 \cdot 24}$ $8 \overline{)4 \cdot {}^{4}2\ 4}$ 0 · $8 \overline{)4 \cdot {}^{4}2\ {}^{2}4}$ 0 · 5 $8 \overline{)4 \cdot {}^{4}2\ {}^{2}4}$ 0 · 5 3

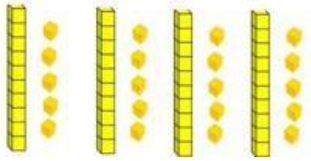
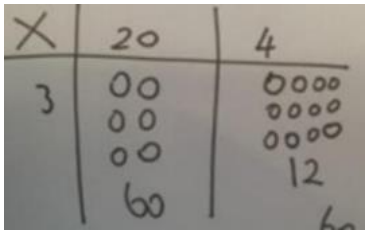
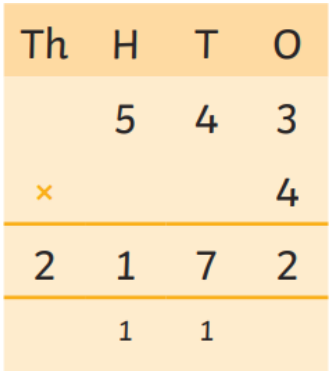
Year 6: Addition

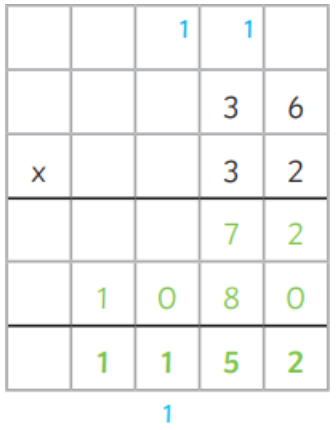
Strategy	Concrete	Pictorial	Abstract
Column addition (compact) <u>with and without</u> regrouping/ exchanging **(addition up to and including five digit numbers).**	**Without regrouping:** Use dienes apparatus to physically add thousands, hundreds, tens and ones. **With regrouping:** **119 + 103 = 222** Physically exchange ten ones for a ten, ten tens for a hundred and ten hundreds for a thousand. 	**Without regrouping:** Draw dienes apparatus and add ones first, then add tens, then add hundreds and finally add thousands. **With regrouping:** Draw dienes apparatus and **to add from the right to the left, beginning with the ones as with compact column addition.** When exchanging, cross out and regroup e.g. Cross out ten ones and add the extra ten into the tens column.	**Without regrouping, one regroup/exchange and multiple regrouping/exchanges:**  Work from the right to the left, beginning with the ones. When exchanges take place, they should be recorded beneath the calculation.
Using the inverse to check calculations and identify missing numbers.	Use practical apparatus such as counters, dienes apparatus, cubes etc. to form addition number sentences and then the related addition sentence using the commutative law and the related subtraction number sentences.	Use pictorial models including bar models and part, whole models to show the inverse operation and the related number sentences. E.g. **53,476 - ? = 20,744** **24,356 = ? + 12,423**   	Use formal methods for column addition and subtraction to demonstrate understanding of the inverse operation. **35,718 + ? = 48,675**  **75,351 – 40,428 =** **The difference between 34,623 and 75,351 is 40,728. Use the inverse to check this statement.**

Year 6: Subtraction

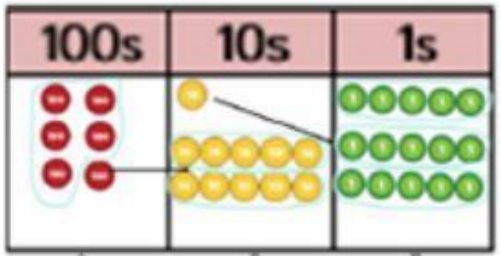
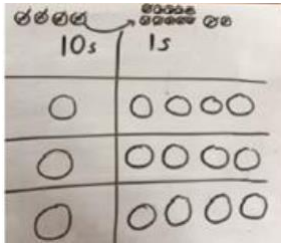
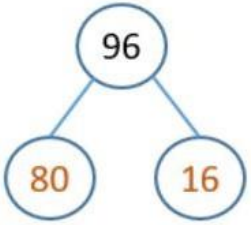
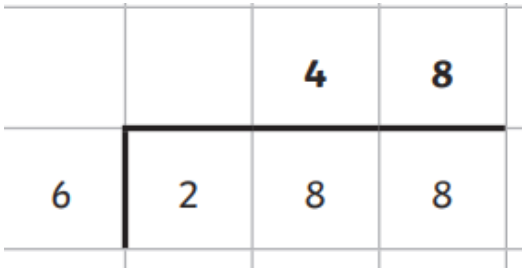
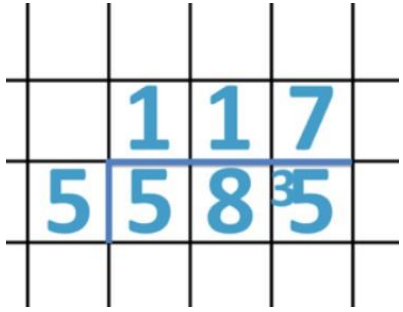
Strategy	Concrete	Pictorial	Abstract
Compact column subtraction <u>with and without</u> exchanging.	**Without exchanging:** **148 – 17 =** Physically take away the ones, then the tens and then the hundreds.  **With exchanging:** **32 – 7 =** Make the largest number using dienes apparatus. Physically take away the ones, then the tens and finally the hundreds. If there are not enough ones, exchange one ten for ten units. If there are not enough tens, exchange one hundred for ten tens.	**Without exchanging:** Draw the largest numbers. Cross out the ones being taken away, followed by the tens and then the hundreds. **With exchanging:** **47 – 19 =** Draw the largest numbers.  If there are not enough ones, exchange one ten for ten units. If there are not enough tens, exchange one hundred for ten tens. Cross out the ones being taken away followed by the tens and the units. 	**With and without exchanging:**  
Finding the difference.	Use practical apparatus to show the difference between two numbers. Equipment such as multilink, which is equal in size and can be lined up exactly, demonstrates this concept.	Use bar models to show finding the difference between two numbers. What is the difference between 25, 567 and 1,875?  	**Number Sentence:** What is the difference between 102,616 and 14,504? **102,616 – 14,504 =**

Year 6: Multiplication

Strategy	Concrete	Pictorial	Abstract
Short Multiplication	Use dienes apparatus to make groups. Combine units and tens. Add together to find the total.  **4 x 15 =** **4 x 10 = 40** **4 x 5 = 20** **40 + 20 = 60** Use counters to represent value of digits to multiply in a place value grid. Recombine tens and ones.  **23 x 3 =** **20 x 3 = 60** **3 x 3 = 9** **60 + 9 = 69**	Draw dienes apparatus or counters to represent place value of digits in columns. **24 x 3 =** **60 + 12 = 72**  	**Short Multiplication:**  Multiply from the right to the left (ones, tens and then hundreds). When exchanges take place, they should be recorded beneath the calculation.

Long Multiplication	***NB: CPA understanding must be in place for short multiplication in order to progress to long multiplication.***	**Long Multiplication**  Begin long multiplication with the ones. Multiply 2 x 6. Write the answer down correctly, recording any exchanges above the calculation in the correct column. 6 x 2 = 12 so place the 2 in the ones column and carry the ten above the calculation. Multiply 2 x 3 (tens). Write the answer down correctly, recording any exchanges above the correct column. 3 x 2 = 6 + 1 ten = 7 tens. Place a zero in the row below in the ones column as the next step requires multiplying by 10. Multiply 3 (tens) x 6. Write the answer down correctly recording any exchanges above the calculation in the correct column. 3 (tens) x 6 = 18. Place the 8 in the tens column and carry the 1 (hundred) into the hundreds column. Multiply 3 (tens) by 3 (tens). Write the answer down correctly recording any exchanges above the calculation in the correct column. 3 (tens) x 3 (tens) = 9. Add 9 to the 1 in the hundreds column and record in the thousands column. Add 1082 + 72. Record any exchanges beneath the calculation.

Year 6: Division

Strategy	Concrete	Pictorial	Abstract
Short Division	**615 ÷ 5 = 123**  Make 615 with place value counters. How many groups of 5 hundreds can you make with 6 hundred counters? Exchange 1 hundred for 10 tens. How many groups of 5 tens can you make with 11 counters? Exchange 1 ten for 10 ones. How many groups of 5 ones can you make with 15 ones?	**Place Value Grid/Part-Whole Model** **42 ÷ 3 = 14**  **96 ÷ 4 = 24**  $(80 \div 4) + (16 \div 4)$ 20 + 4 = 24	**Number Sentence:** Without carrying:  With carrying:  How many 5's in 5 (hundreds)? How many 5's in 8 (tens)? Exchange the remaining 3 tens. How many 5's in 35? ***Children will be required to express remainders as fractions or decimals.***

Long Division	***NB: CPA understanding must be in place for short division in order to progress to long division.***